**CSC 3110**

**Algorithm Design and Analysis**

(30 points)

Due: 04/17/2024 11:59 PM﻿﻿﻿

**Note:** **Submit answers in PDF document format. Please read the submission format for appropriate file naming conventions.**

1. Exercises 9.1:
   1. Problem 1 (2 points)

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Given coins denominations as an array for input (organized by value, highest value starting at position 0), and an integer as the input for the number desired, start at 0, and divide the total to be returned by the largest denomination until the amount remaining is less than the value of the denomination, repeat for all denominations until value to be dispersed is zero. This should give a linear efficiency, as each value of the array needs to only be considered a few times and there should be no recursion.

* 1. Problem 7 (2 points)  
     A close up of a text

     Description automatically generated

This seems fairly straight forward – because everyone has a different rumor, everyone must send at least one message. If everyone sends their message to one person, and that person then messages to everyone the minimum number of messages will be 2n-2 because the person acting as the central distributor does not need to send a message to himself in the two rounds of message sending necessary.   
A whiteboard with arrows and numbers

Description automatically generated

1. Exercises 9.2:
   1. Problem 3 (2 points)   
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      Kruskal’s finds the minimum spanning tree for whatever vertex you start with, the solution therefore is obvious: The algorithm must be adopted to build multiple trees instead, just run it multiple times while iterating through each of the vertices and it will build the minimum spanning tree for each.
   2. Problem 4 (1 point)  
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      Description automatically generated

Yes.

1. Exercises 9.3:
   1. Problem 2 (part a & b) (2 points)   
      A diagram of a hexagon with lines and dots

      Description automatically generated  
      A diagram of a triangle with numbers and circles

      Description automatically generated  
      A whiteboard with a diagram and numbers

      Description automatically generated with medium confidence
   2. Problem 5 (2 points)

A close up of text

Description automatically generated  
Just write Djistra’s algorithm but neglect to write down the sources or save the tree for how they got there.

1. Exercises 9.4:
   1. Problem 3 (part a & b) (2 points)   
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      a.) Codewords of the two least frequent symbols will always have the same length as they will always be on the same level of the tree (the bottom). This is only true if there are two though, if you start getting into odd numbers it won’t work because of the binary tree nature of Huffman encoding.

b.)  
Yes, the nature of Huffman encoding is that frequent symbols are placed higher in the tree. It’s literally how the tree is built – it means that less probably characters are always on the same level of lower – meaning they will always have a longer or equal length.

* 1. Problem 4 (2 points)  
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     Description automatically generated

Huffman trees can suffer from the same issues as most binary trees where all of the symbols end up on one side of the tree. However, the least frequent characters will be on the same level, meaning that the overall result is n-1.

1. Exercises 10.1
   1. Problem 3 (parts a – c) (2 points)  
      A screenshot of a math problem

      Description automatically generated  
      A graph of a function

      Description automatically generated
      1. 3,1 is a unique optimal solution as a minimization function will attempt to reduce the distance from 0,0 as much as possible for x and y. 3,1 is the closest point to zero in terms of distance out of any point on the line, and as such has the unique minimum. C1 = 3 and C2 =5 grant a line that runs along this line while touching no other points.
      2. There are infinitely many optimal solutions when the line described involves the regions boundary. C1 = 1 and C2 = 3 would give such a line.
      3. There are no solutions for C1 = -1, C2 = -1, as they can never be made to met the minimization criteria.
   2. Problem 5 (part a & b) (2 points)  
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      A math equations on a white background

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      A white board with red text and numbers

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      A white paper with blue writing on it

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2. Exercises 10.2
   1. Problem 1 (2 points)  
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      Description automatically generated  
      Based on the definition, a source can have no negative elements in it same row otherwise there would be material flowing TO it, similarly a sink can have no material flowing FROM it. A quick check would be to follow the row, and if all the numbers are positive it is a source, if all the numbers are negative, it is a sink. However, the entire adjacency matrix would have to be scanned, meaning that it would take n^2 time to run.
   2. Problem 3 (part a & b) (2 points)  
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      Description automatically generated  
      a.) There can be more than one optimal solutions for a maximum flow problem even in a network with different capacities on all edges as what matters is that the maximum capacity of a path is the same.

b.)There can be more than one optimal solution to the minimum cut problem for the same reason, because even if the edges are distinct what matters is the values along the entire path.

1. Exercises 10.3
   1. Problem 2 (1 point)  
      A diagram of a network

      Description automatically generated  
      A screenshot of a test

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   2. Problem 3 (part a & b) (2 points)  
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      Description automatically generated  
      a.)Cardinality means how many connections between each set, The smallest is 1 (because there must be at least one match between each set), and the maximum is when all vertices of a set match with all other vertices of the other set. In which case the maximum would be equal to the number of edges.

b.) n! is the largest distinct solution as that represents every vertex having a connection to another vertex in the opposite set without having overlap (factorial is used for combinations). The smallest number of distinct matchings however is 1, where only one in each set matches to only one other in each set.

1. Exercises 10.4
   1. Problem 4 (2 points)  
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      Stability checks are when a man and woman prefer each other over their current partners.

A paper with red check marks and numbers

Description automatically generatedA white sheet with red and blue check marks

Description automatically generated

No parties prefer each other over their current partner in this match up.

* 1. Problem 6 (2 points)  
     

The stable marriage problem does not allow ties, this means that a solution cannot have a parallel ‘also correct’ solution, someone must be better and someone must be worse in preference sorting. As a result every solution is unique. This is also true for woman-optimal problems as it’s the same problem just starting with woman instead.